hep-th/0411185

Pre-Big Bang Scenario on Self-T-Dual Bouncing Branes

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Abstract

We consider a new class of 5-dimensional dilatonic actions which are invariant under T-duality transformations along three compact coordinates, provided that an appropriate potential is chosen. We show that the invariance remains when we add a boundary term corresponding to a moving 3-brane, and we study the effects of the T-duality symmetry on the brane cosmological equations. We find that T-duality transformations in the bulk induce scale factor duality on the brane, together with a change of sign of the pressure of the brane cosmological matter. However, in a remarkable analogy with the Pre-Big Bang scenario, the cosmological equations are unchanged. Finally, we propose a model where the dual phases are connected through a scattering of the brane induced by an effective potential. We show how this model can realise a smooth, non-singular transition between a pre-Big Bang superinflationary Universe and a post-Big Bang accelerating Universe.

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1 Introduction

Two of the main lines of investigation in string cosmology are the Pre-Big Bang scenario and brane cosmology. The former is essentially based on a large symmetry group of the effective bosonic string action which has been studied for many years, leading to very important theoretical and phenomenological results (for a review, see [1]). Brane cosmology is a more recent idea which springs from the pioneering works of Hořava-Witten [2] and Randall-Sundrum [3]-[4]. In the simplest models, our Universe is seen as a warped brane embedded in 5-dimensional space-time, where matter and all interactions except gravity are confined (for reviews, see for example [5]-[7]).

In this paper, we aim to explore possible connections between these two ideas. A first investigation was carried on by one of the authors, in the context of type IIA and type IIB supergravity [8]. These theories are considered as different limits of a unique underlying theory, and solutions of one can be mapped into solutions of the other by T-duality transformations of the fields (see, for example, [9]). This symmetry also holds when the actions are compactified to 5 dimensions, at least in the case considered in [8]. The dual 5-dimensional backgrounds were chosen in order to study a moving brane with a homogeneous and isotropic induced metric. It was found that T-duality transformations of the backgrounds induce the inversion of the brane scale factor. However, provided that the brane Lagrangean is assumed to have a certain form, the cosmological equations are unchanged, which suggests a possible analogy with the Pre-Big Bang scenario. Indeed, in the simplest string cosmological models, the action is invariant under T-duality transformations. Usually, these take the form of an inversion of the scale factor ("scale factor duality") together with a change of sign of the pressure of the cosmic matter, which leave the equations of motion unchanged [1]. This analogy, however, is only valid at the level of the equations of motion. Indeed, in the Pre-Big Bang scenario, the T-duality symmetry group also leaves the action unchanged. However, in the case studied in [8], while the brane equations of motion are invariant under T-duality, the action transforms from type IIA to type IIB (or vice versa).

In light of these results, it is natural to ask what happens when the brane moves in a background which is a solution of an action invariant under T-duality. In this paper, we address this question by first finding such backgrounds. This is not as easy as it might appear, because, together with a self-T dual action, we also need a background such that the induced metric on the brane is homogeneous and isotropic. In Sec. 2, we present a dilatonic action and a metric which meet these requirements. These solutions appear to be new and interesting not only in the context of brane cosmology. In this Section we consider one simple solution to the bulk equations of motion, but we believe that more general ones can be found.

In Sec. 3, we study the Israel junction conditions which arise when the 3-brane is embedded in the bulk. We first show that the addition of the brane action does not spoil the T-duality invariance of the total action. Then, we obtain the brane cosmological equations and we show that T-duality transformations in the bulk induce the inversion of the scale factor on the brane together with a change of sign of the pressure. By assuming a specific form for the brane-matter

Lagrangean, we also show that by choosing an appropriate conformal frame on the brane, the energy is conserved.

In Sec. 4, we consider the cosmological equations in the case when the bulk background is described by the solutions found in Sec. 2. Even in this simple case, we will see that it is not possible to find an exact solution, mainly because the equation of state relating energy density and pressure is manifestly dependent on the position of the brane. However, we will be able to show that, at least in the regimes of late and early times, we recover the main features of most brane cosmological models.

In Sec. 5, we consider a case where the brane scatters against the zero of an effective potential. This model not only strengthens the similarities to the Pre-Big Bang scenario, but also offers a natural interpretation of the dual solutions to the cosmological equations. By assuming that the dual transition takes place at the bounce, we show that there is a transition from a superinflationary phase to a post-big bang phase, which appears to be smooth and non-singular. Finally, we conclude with some remarks and open problems.

2 Self-T-Dual Backgrounds

In this Section we introduce a new family of tensor-scalar actions which show a non-trivial invariance under field transformations. To begin with, consider a 5-dimensional pseudo-Riemannian manifold $\mathcal{M}$ (the bulk space-time), equipped with a metric tensor g_{AB} whose line element reads

$$ds^2 = g_{AB}\, dx^A dx^B = -A^2(r)\, dt^2 + B^2(r)\, dr^2 + R^2(r)\, \delta_{ij}\, dx^i\, dx^j. \tag{2.1}$$

We assume that the functions A, B and R of the radial coordinate r satisfy the equations of motion derived from the bulk tensor-scalar action

$$S_{\text{bulk}} = \int_{\mathcal{M}} d^5x \sqrt{g}\, e^{-2\phi} \left[\mathcal{R} + 4(\nabla\phi)^2 + V\right], \tag{2.2}$$

where $\mathcal{R}$ is the Ricci scalar, ϕ is the dilaton field, and the potential V is some function of ϕ and possibly g_{AB}. Explicit solutions to the equations of motion are well-known in the case when the dilaton is a function of r only and the potential has the Liouville form $V(\phi) = V_0\, e^{k\phi}$ [10].

We shall now show that another class of solutions exists provided the potential is a function of the so-called shifted dilaton[3], defined as

$$\bar{\phi}(r) = \phi(r) - \frac{3}{2}\log R(r). \tag{2.3}$$

This definition holds provided that the volume of the spatial sections is constant at each fixed r [1]. With such a potential, and a line element of the form (2.1), the action has a non-trivial symmetry which can be exploited to generate a new and inequivalent solution to the equations

[3] In the context of string cosmology, this kind of potential is often called "non-local" [1].

of motion from a known one. To show this, we compute $\mathcal{R}$ in terms of the fields A, B, R and ϕ, and write the action as

$$S_{\text{bulk}} = -2 \int_{\mathcal{M}} d^5x \frac{AR^3 e^{-2\phi}}{B} \left[3\mathcal{H} \left(\frac{A'}{A} - \frac{B'}{B} \right) - \frac{A'B'}{AB} + 6\mathcal{H}^2 \right. \tag{2.4}$$

$$\left. + \frac{A''}{A} + 3\mathcal{H}' - 2(\phi')^2 - \frac{B^2 V}{2} \right],$$

where the prime stands for differentiation with respect to r, and we have defined $\mathcal{H} := R'/R$. We can now change from ϕ to $\bar{\phi}$ and eliminate the two second derivatives $\mathcal{H}'$ and A'' by using the identities

$$\begin{aligned} \frac{Ae^{-2\bar{\phi}}}{B} \left[\mathcal{H} \left(\frac{A'}{A} - \frac{B'}{B} \right) + \mathcal{H}' - 2\phi'\mathcal{H} + 3\mathcal{H}^2 \right] &= \frac{d}{dr} \left(\frac{A\mathcal{H}e^{-2\bar{\phi}}}{B} \right), \\ \frac{A'e^{-2\bar{\phi}}}{B} \left(\frac{A''}{A'} - \frac{B'}{B} - 2\bar{\phi}' \right) &= \frac{d}{dr} \left(\frac{e^{-2\bar{\phi}}A'}{B} \right). \end{aligned} \tag{2.5}$$

We can thus use Stokes' theorem, and the action reads

$$\begin{aligned} S_{\text{bulk}} &= \int_{\mathcal{M}} d^5x \, e^{-2\bar{\phi}} \left[ABV(\bar{\phi}) - \frac{3A\mathcal{H}^2}{B} + \frac{4(\bar{\phi}')^2 A}{B} - \frac{4\bar{\phi}' A'}{B} \right] \\ &\quad -2 \int_{\partial\mathcal{M}} d^3x \, dt \, e^{-2\bar{\phi}} \left(\frac{3A\mathcal{H}}{B} + \frac{A'}{B} \right). \end{aligned} \tag{2.6}$$

Note that we have also assumed that the potential V, which in principle depends on both ϕ and g_{AB}, becomes a function of *only* the shifted dilaton $\bar{\phi}$ under the metric ansatz (2.1). Since the fields A, B, and R are independent of all coordinates except r, the metric (2.1) and the equations of motion obtained from the action (2.2) are clearly invariant under translations along the compact coordinates x^i, $i = 1, 2, 3$. In addition, if we assume that the fields vanish at the boundary $\partial\mathcal{M}$, the action (2.6) is invariant under the field transformations

$$R(r) \xrightarrow{T} \tilde{R}(r) = R(r)^{-1} \quad \Rightarrow \quad \mathcal{H} \to -\mathcal{H} \tag{2.7}$$

$$\phi(r) \xrightarrow{T} \tilde{\phi}(r) = \phi(r) - 3\log R(r) \quad \Rightarrow \quad \bar{\phi}(r) \to \bar{\phi}(r). \tag{2.8}$$

Therefore, for every solution to the equations of motion with line element (2.1) and dilaton $\phi(r)$, there exists a dual counterpart with

$$d\tilde{s}^2 = -A^2(r)dt^2 + B^2(r)dr^2 + R^{-2}(r)\delta_{ij}\, dx^i dx^j, \tag{2.9}$$

and dilaton $\tilde{\phi}$. In the context of string theory, this is the well known T-duality symmetry and the transformations (2.7) and (2.8) are a particular case of Buscher's transformations [11, 12] applied along each x^i.

The equations of motion, obtained by variation of (2.6) with respect to the fields A, B, R and $\bar{\phi}$, read

$$\begin{aligned}
\bar{\phi}'' - \bar{\phi}' \left(\frac{A'}{A} - \frac{B'}{B} \right) - \frac{3}{2}\mathcal{H}^2 &= 0 && (2.10) \\
\bar{\phi}'' - \bar{\phi}' \frac{\mathcal{H}'}{\mathcal{H}} + \frac{1}{2} B^2 V(\bar{\phi}) &= 0 && (2.11) \\
\frac{d}{dr}\left[\log\left(\frac{A\mathcal{H}e^{-2\bar{\phi}}}{B} \right) \right] &= 0 && (2.12) \\
\frac{\partial V(\bar{\phi})}{\partial \bar{\phi}} - 4V(\bar{\phi}) - \frac{4\mathcal{H}}{B^2}\frac{d}{dr}\left[\frac{1}{\mathcal{H}} \left(\frac{\mathcal{H}'}{\mathcal{H}} - \frac{B'}{B} \right) \right] &= 0. && (2.13)
\end{aligned}$$

However, it can be shown that if $\bar{\phi}' \neq 0$, the last of these equations is just a combination of the other three. Therefore we are left with three equations for the five unknown functions A, B, R, V and $\bar{\phi}$. If one defines new functions $N(r)$ and $P(r)$ by

$$N := \frac{Ae^{-2\bar{\phi}}}{B}, \quad P := \frac{1}{AB}, \tag{2.14}$$

then (2.10)-(2.12) can be rewritten as

$$\begin{aligned}
\bar{\phi}'' + \frac{P'}{P}\bar{\phi}' - \frac{3}{2}\mathcal{H}^2 &= 0, && (2.15) \\
N\mathcal{H} &= k, && (2.16) \\
\bar{\phi}'' - \frac{\mathcal{H}'}{\mathcal{H}}\bar{\phi}' + \frac{V(\bar{\phi})}{2NP}e^{-2\bar{\phi}} &= 0, && (2.17)
\end{aligned}$$

where k is an arbitrary constant.

We found a large number of singular and non-singular solutions to these equations, which often display an unusual asymptotic structure, i.e. they are neither asymptotically flat nor de Sitter or anti-de Sitter. These will be discussed in detail elsewhere [13]. For the moment, we consider a simple solution which will be useful later: if we assume that the shifted dilaton has the form

$$\bar{\phi} = \alpha \ln r - \beta \ln P, \tag{2.18}$$

and that $R(r) = r$, then Eq. (2.15) yields

$$P'' - \frac{\alpha}{\beta r}P' + \frac{(2\alpha + 3)}{2\beta r^2}P = 0. \tag{2.19}$$

The general solution to this is

$$P(r) = p_+ r^{m_+} + p_- r^{m_-}, \tag{2.20}$$

where α, β, p_+ and p_- are arbitrary constants, and

$$m_\pm = \frac{1}{2\beta}\left[\alpha + \beta \pm \sqrt{(\alpha-\beta)^2 - 6\beta}\right]. \tag{2.21}$$

Then, it follows that

$$A^2 = k\, r^{\,2\alpha+1} P^{\,-2\beta-1}, \tag{2.22}$$

$$B^2 = k^{-1} r^{\,-2\alpha-1} P^{\,2\beta-1}, \tag{2.23}$$

$$V(\bar\phi) = \frac{2kp_+p_-}{\beta}\left[(\alpha-\beta)^2 - 6\beta\right] e^{\frac{(2\beta-1)}{\beta}\,\bar\phi}, \tag{2.24}$$

$$\phi = \left(\alpha + \frac{3}{2}\right)\ln r - \beta \ln P. \tag{2.25}$$

The T-duality symmetry of the action is manifest in the equations of motion (2.15)-(2.17), which are indeed invariant under the transformations[4] (2.7) and (2.8). Hence, to the solution above it corresponds a new and inequivalent one obtained by replacing $R = r$ with $\tilde{R} = 1/r$ and ϕ with $\phi - 3\ln r$.

3 T-Duality and Junction Conditions

We now investigate the properties of a 3-brane Σ embedded in the bulk space-time $\mathcal{M}$. In particular, we focus on a moving 3-brane with an induced metric of the form

$$ds^2_\Sigma = -d\tau^2 + R^2(r)\,\delta_{ij}\,dx^i\,dx^j, \tag{3.1}$$

where the cosmic time τ will be defined shortly. Similar models have been widely studied, and it is well-known that an observer living on the moving brane will experience an evolving 4-dimensional Universe [5]-[7]. In particular, we are interested on the effects that the T-duality symmetry of the bulk might have on the brane cosmological equations.

The presence of the 3-brane introduces the extra term in the action

$$S_{\text{brane}} = -\int_\Sigma d^3x\,d\tau\sqrt{h}e^{-2\phi}\left[2\left(K^+ + K^-\right) + \mathcal{L}\right], \tag{3.2}$$

where $K^\pm$ are the extrinsic curvatures on the two sides of the brane. $\mathcal{L}$ represents the Lagrangean of the matter confined on the brane and h is the determinant of the induced metric. When the bulk metric is of the form (2.1), then $\sqrt{h} = R^3$ and we have the relation $\sqrt{h}\,e^{-2\phi} = e^{-2\bar\phi}$.

[4]Note that, under the transformation (2.7), we also have that $k \to -k$ in Eq. (2.16). However, given that k is arbitrary, we shall ignore this detail.

If we assume a $\mathbb{Z}_2$ symmetry around the brane then $K^+ = K^- = K$, and the junction conditions (in the string frame [8, 14]) read

$$2K_{\mu\nu} = T_{\mu\nu} - \frac{1}{2}T^\phi h_{\mu\nu}, \tag{3.3}$$

$$4n^A\nabla_A\phi = T - \frac{3}{2}T^\phi. \tag{3.4}$$

In these equations, $T_{\mu\nu}$ defines the energy-momentum tensor of the brane matter

$$T_{\mu\nu} = -\frac{1}{\sqrt{h}}\frac{\delta(\sqrt{h}\mathcal{L})}{\delta h^{\mu\nu}}, \tag{3.5}$$

T^ϕ is the variation of the Lagrangean with respect to the scalar field

$$T^\phi = -\frac{1}{2}e^{2\phi}\,\frac{\delta(e^{-2\phi}\mathcal{L})}{\delta\phi}, \tag{3.6}$$

and n^A is the unit vector normal to the brane pointing into the bulk[5]. If the bulk and induced metrics are (2.1) and (3.1) respectively, then the conformal time is implicitly defined by the normalisation condition

$$A\dot{t} = \sqrt{1 + B^2\dot{r}^2}. \tag{3.7}$$

(Choosing the positive, rather than the negative, square root here ensures that the conformal time τ increases with the bulk coordinate time t, i.e. that the unit normal vector points *into* the bulk [15].) With these conventions, the components of the normal vector and of the extrinsic curvature read [16]

$$n_t = AB\dot{r}, \quad n_r = -B\sqrt{1 + B^2\dot{r}^2}, \tag{3.8}$$

$$K_{ij} = -\frac{R'}{BR}\sqrt{1 + B^2\dot{r}^2}\,h_{ij}, \quad K_{\tau\tau} = \frac{1}{AB}\frac{d}{dr}\left(A\sqrt{1 + B^2\dot{r}^2}\right). \tag{3.9}$$

Normally, the conditions (3.3) and (3.4) give the Friedmann equation on the brane, together with an energy (non-)conservation equation and a junction condition for the dilaton [5]-[7]. The question is how these equations behave under T-duality transformations in the bulk. The main motivation is that the transformations (2.7) implies

$$h_{ij} \xrightarrow{T} \tilde{h}_{ij} = \frac{1}{R^4(r)}h_{ij}, \tag{3.10}$$

which means that the scale factor of the induced metric undergoes inversion under T-duality. In the context of string cosmology, this transformation is known as scale factor duality and it leaves the cosmological equations unchanged, provided that $p \to -p$, where p is the pressure of

[5]Here we adapt the conventions of [14], where the dilaton coupling in the action is $\exp(-\phi)$ instead of $\exp(-2\phi)$.

the matter [1]. In our case, the junction conditions (and hence the cosmological equations) are not invariant under T-duality, unless some transformation rules are assumed for the terms $T_{\mu\nu}$ and T^{ϕ}. Indeed, given that under T-duality $\phi \to \phi - 3\ln R$, the second junction condition will contain an extra term in the derivative. Moreover, the ij components of the extrinsic curvature transform according to

$$K_{ij} \xrightarrow{T} \tilde{K}_{ij} = -\frac{1}{R^4(r)} K_{ij}, \tag{3.11}$$

hence the right hand side of the ij component of the first junction condition changes sign.

One can argue that when we add the bulk action to the term (3.2), we lose T-duality invariance. Indeed, given the components (3.9) of $K_{\mu\nu}$, we see that the trace K which appears in the brane action transforms as

$$K \xrightarrow{T} \tilde{K} = K - 2K_{ij}h^{ij}. \tag{3.12}$$

On the contrary, we can show that the *total* action is in fact T-duality invariant. To do so, we recall that the bulk action was reduced to the form (2.6), and we assumed that the second integral, i.e. the boundary term, vanished on $\partial\mathcal{M}$. However, the presence of Σ results in an additional boundary for the bulk space, and the boundary term will not in general vanish at the location of the brane. We therefore have to add its contribution to (3.2). Therefore, the sum of *all* boundary terms becomes

$$S_{\text{boundary}} = -2\int_{\partial\mathcal{M}} d^3x\, dt\, e^{-2\bar{\phi}} \left[\frac{3AR'}{BR} + \frac{A'}{B}\right] - \int_{\Sigma} d^3x\, d\tau\, e^{-2\bar{\phi}} \left[2(K^+ + K^-) + \mathcal{L}\right]. \tag{3.13}$$

Recall that $\partial\mathcal{M} = \partial\mathcal{M}_+ \cup \partial\mathcal{M}_-$, where $\partial\mathcal{M}_\pm$ are the boundaries of the bulk on either side of the brane. As submanifolds of $\mathcal{M}$, these are both equivalent to the brane Σ, but are oriented in opposite directions (one with normal vector n^A, the other with $-n^A$). The $\mathbb{Z}_2$ symmetry, however, identifies these two boundaries with each other, meaning that $\int_{\partial\mathcal{M}} = 2\int_\Sigma$. Thus, by writing

$$dt = \frac{dt}{d\tau} d\tau = \dot{t}\, d\tau, \tag{3.14}$$

by using the normalization condition (3.7), and the components of the extrinsic curvature, we find

$$S_{\text{boundary}} = -\int_{\Sigma} d^3x\, d\tau\, e^{-2\bar{\phi}} \left[\mathcal{L} + \frac{4}{B}\frac{d}{dr}(A\dot{t})\right]. \tag{3.15}$$

This term, and hence the total action $S = S_{\text{bulk}} + S_{\text{brane}}$, is invariant under the transformations (2.7) and (2.8), provided that

$$\mathcal{L} \xrightarrow{T} \tilde{\mathcal{L}} = \mathcal{L}. \tag{3.16}$$

Despite its simple form, this transformation is far from obvious, because $\mathcal{L}$ could be a complicated function of the matter fields, the induced metric and the dilaton. However, some insight can be obtained by imposing the invariance of the junction conditions and studying how the brane-matter energy-momentum tensor behaves.

To start with, we first assume that the energy-momentum tensor for the brane matter has the form $T^{\mu}{}_{\nu} = diag(-\mu, p, p, p)$. Then we use Eq. (3.3) to obtain the expression

$$\frac{3}{2}T^{\phi} = -2K_{ij}h^{ij} + T_{ij}h^{ij}, \tag{3.17}$$

which we insert into Eq. (3.4), obtaining

$$4n^A\nabla_A(\bar{\phi} + \frac{3}{2}\ln R) = T + 2K_{ij}h^{ij} - T_{ij}h^{ij}. \tag{3.18}$$

By using Eqs. (3.8)-(3.9), we find that

$$K_{ij}h^{ij} = 3n^A\nabla_A(\ln R), \tag{3.19}$$

and the junction condition (3.4) reads

$$4n^A\nabla_A\bar{\phi} = -\mu. \tag{3.20}$$

By defining the "shifted" pressure

$$\bar{p} := p - \frac{1}{2}T^{\phi}, \tag{3.21}$$

we can write the independent components of the junction conditions (3.3) as

$$2K_{ij} = \bar{p}\,h_{ij}, \tag{3.22}$$

$$2K_{\tau\tau} = \mu + \frac{T^{\phi}}{2}, \tag{3.23}$$

Finally, by inserting the expressions for the components of the extrinsic curvature into these equations, by using the equation of motion (2.10), and by defining ω and the Hubble "constant" H by

$$\omega := -\frac{R'}{2R\bar{\phi}'}, \qquad H := \frac{\dot{R}}{R} \tag{3.24}$$

(where the dot stands for differentiation with respect to the cosmic time τ), we find that the junction conditions reduce to the three independent equations

$$\bar{p} = \omega\mu, \tag{3.25}$$

$$H^2 = \frac{(\omega\mu)^2}{4} - \left(\frac{R'}{RB}\right)^2, \tag{3.26}$$

$$\dot{\mu} + 3H\bar{p} = \dot{\bar{\phi}}\left(2\mu + T^{\phi}\right). \tag{3.27}$$

The first is the effective equation of state for a perfect fluid confined on the brane and the second is similar to the usual Friedmann equation. Finally, the third reveals that the energy on the brane is not in general conserved, as normally happens in brane cosmology whenever there is a bulk dilaton field [5]-[7]. Note that under the transformation (2.7), the equation of state is “reflected”, i.e.

$$\omega \xrightarrow{T} \tilde{\omega} = -\omega. \tag{3.28}$$

This proves that the cosmological equations on the brane are manifestly T-duality invariant, provided that

$$\mu \xrightarrow{T} \tilde{\mu} = \mu, \qquad \bar{p} \xrightarrow{T} \tilde{\bar{p}} = -\bar{p} \quad \Leftrightarrow \quad \omega \xrightarrow{T} \tilde{\omega} = -\omega. \tag{3.29}$$

In string cosmology, the scale factor duality is always followed by the reflection of the equation of state, which is required by the $O(d,d)$ invariance of the action when a matter Lagrangean describing a perfect fluid is included [17]. Therefore, by imposing T-duality invariance on the junction conditions, we obtain a brane cosmological model which shares the essential features of the Pre-Big Bang scenario.

The conditions (3.29) can be clarified by choosing a specific form for $\mathcal{L}$. Following [18], we assume that

$$\mathcal{L} = f^2(\phi) z(\phi) L(\psi, \nabla\psi, \gamma_{\mu\nu}), \tag{3.30}$$

where ψ represents generic fields living on the brane which couple to the bulk dilaton only through a conformal metric $\gamma_{\mu\nu} = f(\phi) h_{\mu\nu}$. Hence, we can define the energy-momentum tensor with respect to the conformal metric $\gamma_{\mu\nu}$ as

$$S_{\mu\nu} = -\frac{1}{\sqrt{\gamma}} \frac{\delta(\sqrt{\gamma}\, L)}{\delta \gamma^{\mu\nu}}, \tag{3.31}$$

where $\sqrt{\gamma} = f^2(\phi)\sqrt{h}$. The two energy-momentum tensors are then related by

$$T_{\mu\nu} = f(\phi) z(\phi) S_{\mu\nu}, \tag{3.32}$$

and if we set $S^{\mu}{}_{\nu} = diag(-\rho, \pi, \pi, \pi)$, we obtain

$$\mu = f^2(\phi) z(\phi) \rho, \qquad p = f^2(\phi) z(\phi) \pi. \tag{3.33}$$

In particular, with the choice $z(\phi) = e^{2\phi}$, we find that

$$T^{\phi} = -\frac{1}{2} f(\phi) \frac{df(\phi)}{d\phi}\, e^{2\phi} S^{\mu}{}_{\mu}. \tag{3.34}$$

Thus, by using Eq. (3.32), we obtain the relation between T^{ϕ} and the trace of the energy-momentum tensor T

$$T^{\phi} = -\frac{T}{2} \frac{d}{d\phi} \ln f(\phi). \tag{3.35}$$

Incidentally, this formula can be used to show that, in the conformal frame defined by $\gamma_{\mu\nu}$, the energy on the brane is conserved. In terms of p and ϕ, Eq. (3.27) reads

$$\dot{\mu} + 3H(p + \mu) = \dot{\phi}(2\mu + T^{\phi}). \tag{3.36}$$

Let the line element corresponding to the metric $\gamma_{\mu\nu}$ be

$$ds_\gamma^2 = -d\xi^2 + E^2(r,\phi)\delta_{ij}\,dx^i dx^j = f(\phi)(-d\tau^2 + R^2(r)\delta_{ij}\,dx^i dx^j). \tag{3.37}$$

Thus, $d\xi = \sqrt{f(\phi)}d\tau$ and $E(r,\phi) = \sqrt{f(\phi)}R(r)$. By using Eq. (3.33), we find that the conservation equation reads

$$\dot\rho + 3\frac{\dot E}{E}(\rho+\pi) = 2\dot\phi - \frac{\dot z}{z}\rho, \tag{3.38}$$

where now the dot stands for differentiation with respect to the conformal time ξ. Therefore, when we set $z(\phi) = e^{2\phi}$, the above equation reduces to

$$\dot\rho + 3\frac{\dot E}{E}(\rho+\pi) = 0, \tag{3.39}$$

which shows that, in the conformal frame defined by $\gamma_{\mu\nu}$, the energy on the brane is conserved.[6]

We now come back to the junction conditions (3.20), (3.22) and (3.23). First, we note that Eq. (3.20) implies that, no matter what form of $\mathcal{L}$ we choose, under T-duality we must have

$$\mu \xrightarrow{T} \tilde\mu = \mu. \tag{3.40}$$

By using Eq. (3.35), we can write Eqs. (3.22) and (3.23) as

$$\begin{aligned} 2K_{ij} &= \left(1 - \frac{3}{2}\sigma'\right)ph_{ij} + \frac{1}{2}\mu\sigma' h_{ij} && (3.41)\\ 2K_{\tau\tau} &= \left(1 - \frac{1}{2}\sigma'\right)\mu + \frac{3}{2}\,\sigma' p, && (3.42)\end{aligned}$$

where we set $f(\phi) = e^{-2\sigma(\phi)}$ and where $\sigma' = \frac{d\sigma}{d\phi}$. Given that, under T-duality,

$$K_{ij} \xrightarrow{T} \tilde K_{ij} = -\frac{1}{R^4}K_{ij}, \qquad h_{ij} \xrightarrow{T} \tilde h_{ij} = \frac{1}{R(r)^4}h_{ij}, \tag{3.43}$$

we see that in order to preserve T-duality invariance of the junction conditions, $\sigma(\phi)$ and p would have to transform in a (possibly very) complicated way. But an alternative would be to simply require $\sigma' = 0$ and $p \xrightarrow{T} \tilde p = -p$. In this case, the matter on the brane is coupled to the bulk dilaton only through the factor $e^{-2\phi}$ which appears in the brane action (3.2).

The choice $\sigma' = 0$ might seem to be something of a trivial case, but we now show that it still leads to a very interesting cosmological model.

[6] Note that, in analogy with our results, the energy on the brane in [18] is conserved only if $z(\phi) = 1$.

4 Self T-Dual Brane Cosmology

In this section we study the brane cosmological equations in the case when the bulk metric and potential are given by Eqs. (2.22), (2.23), and (2.24) respectively. We also have $R(r) = r$, so the shifted dilaton reads

$$\bar{\phi}(r) = \alpha \ln r - \beta \ln P(r), \tag{4.1}$$

where $P(r)$ is given by Eq. (2.20). If we also require σ' to vanish, the junction conditions reduce to

$$p = \omega\mu, \tag{4.2}$$

$$H^2 = \left(\frac{\omega\mu}{2}\right)^2 - \frac{1}{B^2 r^2}, \tag{4.3}$$

$$\dot{\mu} = 2\dot{\bar{\phi}}\mu - 3Hp, \tag{4.4}$$

where $\omega = -(2r\bar{\phi}')^{-1}$. Note also that in the Friedmann equation, H^2 depends on p^2 and not on μ^2, as in the usual brane cosmology. Despite these complications, we can still study these equations in the small or large scale factor regimes, respectively corresponding to the early or late Universe.

Let us begin with a phase of small scale factor. We assume that during this phase there is a time τ at and around which ω is more or less constant (or at least very slowly varying). At this time, H and $\dot{\bar{\phi}}$ are proportional to one another, and we can set $H \simeq \kappa\dot{\bar{\phi}}$, and hence $\omega \simeq -\kappa/2$. Thus the energy conservation equation (4.4) can be integrated, yielding

$$\mu(\tau) \simeq \mu_0 \exp\left[\left(\frac{3}{2}\kappa^2 + 2\right)\bar{\phi}(\tau)\right], \tag{4.5}$$

where μ_0 is an arbitrary constant of integration. By choosing $m_+ > m_-$ in Eq. (2.21), we see that $P \simeq p_- r^{m_-}$ for small r, hence

$$\bar{\phi}' r = -\frac{1}{2\omega} \simeq \alpha - \beta m_-. \tag{4.6}$$

In particular, we can assign to ω the physically relevant value 1/3, which corresponds to a radiation-dominated Universe. In this case, Eq. (4.6) implies that $\alpha = -3/2$ and hence that $m_+ = (\beta - 3/2)/\beta$, $m_- = 0$. Note that when $\alpha = -3/2$, the dilaton field reads $\phi = \bar{\phi} + 3/2 \ln r = -\beta \ln P$.

If instead we assign to α the value $-3/2$ *a priori*, then $P \simeq p_-$ for small r, which is consistent with the above assumptions. The integration of Eq. (4.4) is then straightforward and yields $\mu(\tau) \propto r(\tau)^{-4}$, which describes a radiation-dominated Universe, as in standard cosmology. Finally, the Friedmann equation can be solved and we have the non-standard solution $r(\tau) \propto \tau^{1/4}$, typical of brane cosmology [5]-[7]. To this solution corresponds a T-dual counterpart with the scale factor $R = 1/r$ and $\omega = -1/3$, i.e. a large universe with an equation of state typical of a gas of stretched strings (see references in [1]).

We now the case when the scale factor is large. We assume that this case corresponds to the Universe as seen in the present epoch (i.e. matter-dominated), and we set $\omega \simeq 0$. Therefore, we are left with only two independent equations of motion:

$$2K_{\tau\tau} = \mu, \qquad 4n^A \nabla_A \bar{\phi} = -\mu. \tag{4.7}$$

In particular, the Friedmann equation reads

$$H^2 = \left(\frac{\mu}{4\bar{\phi}' r}\right)^2 - \frac{1}{B^2 r^2}, \tag{4.8}$$

and the anomalous μ^2 dependence is usually treated by splitting the energy density into the sum of a time-dependent term and a constant tension, i.e. by setting $\mu = \mu(\tau) + \mathcal{T}$ [5]-[7]. For $\alpha = -3/2$, we note that $\bar{\phi}' r \simeq -\beta$ at large r. Hence, the energy-conservation equation can be integrated, yielding

$$\mu(\tau) \propto r(\tau)^{-\frac{1}{2\beta}(3+4\beta^2)}, \tag{4.9}$$

and the energy density is decreasing in time for any $\beta > 0$. At large times, the contribution of the squared energy density becomes negligible together with the second term on the right hand side of Eq. (4.8). Hence, the Friedmann equation reduces to

$$H^2 \simeq \frac{\mu \mathcal{T}}{8\beta^2} + \frac{\mathcal{T}^2}{16\beta^2}, \tag{4.10}$$

which describes an expanding, accelerating Universe with positive cosmological constant. Like in the previous case, also this solution has a dual counterpart with small scale factor and vanishing pressure. We now provide for a possible interpretation of these dual phases.

5 Bouncing Branes

In this section, we explore the possibility that the transition between T-dual phases is triggered by the scattering of the brane against a zero of an effective potential. Following [19], we can use the results of the previous Section to define an effective potential $W(r)$ by

$$W := \frac{1}{B^2} - \frac{\mu^2}{16\left(\bar{\phi}'\right)^2} \tag{5.1}$$

and write the Friedmann brane equation in the simple form

$$\dot{r}^2 + W(r) = 0. \tag{5.2}$$

This equation describes a particle moving with velocity $\dot{r}$ in a 1-dimensional potential $W(r)$ whose zeroes corresponds to classical turning points. So let us assume that there exists $r_0 > 0$

such that $W(r_0) = 0$ and $W'(r_0) < 0$; this means that a brane moving in from the region $r > r_0$ with $\dot{r} < 0$ will bounce off the effective potential at $r = r_0$ and move in the direction of increasing r, i.e. $\dot{r} > 0$. We refer to the former as the "pre-bounce" epoch and the latter as the "post-bounce" epoch, with the bounce itself taking place at cosmic time $\tau = 0$. The acceleration $\ddot{r} = -W'/2$ is always positive, at least in the region near r_0.

Note that the effective potential and the position of its zeroes are unchanged by T-duality transformations, so that (5.1) holds for both $\tilde{R}(\tau) = r(\tau)$ and the dual $R(\tau) = 1/r(\tau)$. We can utilise this property to ensure an always-expanding universe: recall that R is the scale factor for the spatial part of our Universe, so $\dot{R}$ is the expansion rate. During the pre-bounce epoch, $\dot{r} < 0$, but if we choose this to also be the dual phase of our bulk, i.e. $\tilde{R} = 1/r$, we see that $\dot{\tilde{R}} = -\dot{r}/r^2 > 0$. If we take the post-bounce epoch as corresponding to the normal phase $R = r$, then $\dot{R} = \dot{r} > 0$. Thus, by requiring that the T-duality phase transition happens when the brane bounces off $r = r_0$, we have an Universe that always expands.

Let us look at the two epochs in a bit more detail:

1. **Pre-Bounce:** Since $\tilde{R} = 1/r$, $\tilde{H} = -\dot{r}/r$ and therefore

$$\dot{\tilde{H}} = -\frac{\ddot{r}}{r} + \frac{\dot{r}^2}{r^2} \quad = \quad \frac{1}{2r}W'(r) - \frac{1}{r^2}W(r). \tag{5.3}$$

   In the region near the bounce, where $W \approx 0$ and $W' < 0$, we have $\dot{\tilde{H}} < 0$, typical of a power-law inflationary scenario. The sign of $\dot{\tilde{H}}$ for large r (i.e. very negative τ), depends on the shape of the potential. However, there is a very large class of potentials $W(r)$ such that $\dot{\tilde{H}}$ can be made positive for large r (e.g. W approaches a negative constant for $r \to \infty$). Such cases imply a superinflationary Universe (i.e. accelerating with increasing curvature) for large negative times.

2. **Post-Bounce:** $R = r$ and $H = \dot{r}/r$, so $\dot{H} = -\dot{\tilde{H}}$. $\dot{H}$ is therefore positive near $r = r_0$, and the curvature is increasing. However, if $W(r)$ is such that $\dot{\tilde{H}} > 0$ for large r, then $\dot{H} < 0$ in the same region, which in the post-bounce scenario corresponds to large positive τ. Hence, at large times, the Universe is accelerating but its curvature is decreasing.

We see that, if the potential $W(r)$ has the right shape, we can have a dual transition between a superinflationary Universe (the pre-bounce epoch) and a post-inflationary, accelerating Universe (the post-bounce epoch), characterized by the inversion of the scale factor and a reflection of the equation of state. Remarkably, these features are also typical of the Pre-Big Bang scenario [1]. Moreover, the transition between the two T-dual phases occurs at a *finite* value of the scale factor, corresponding to $r(0) = r_0$. This is reminiscent of some Pre-Big Bang models, where the presence of a shifted dilaton potential in the action avoids the formation of a singularity at the dual transition [1, 20, 21]. In particular, in our model we can always tune the integration constants in order to set r_0 equal to the self-dual radius, defined as the value of r such that the scale factor and its dual are the same (in our normalization units, this is simply $r_0 = 1$). It thus follows that the Universe must have a minimum size determined by the self-dual radius, which in turn is determined by the location of the zero of the effective potential $W(r)$.

The transition between the dual Universes is characterised by an interesting phase where the superinflation becomes power-law inflation because of the change of sign of $\dot{\tilde{H}}$. Then, at $\tau = 0$, the dual transition occurs, the curvature begins to increase, and the inflationary phase ends. Finally, the curvature starts to decrease again and, at late times, the Universe eventually enters our present epoch of accelerated expansion. Such a behaviour depends entirely on the shape of the effective potential $W(r)$. It easy to show that the approximate solutions considered in Sec. 4 fit in this model for $r \gg r_0$. However, a precise description of all these cosmological phases requires first to find exact solutions of the brane cosmological equations.

6 Conclusions

The results presented in this paper might build a bridge between brane cosmology an Pre-Big bang scenario, and offer new lines of investigations which, we believe, are worth studying. First of all, the tensor-scalar action introduced in Sec. 2 represents a new class of backgrounds which can be much more general than the one considered here. For example, it would be interesting to find black hole solutions to the equations of motion (2.15)-(2.17), and analyze their thermodynamical properties in light of the self-T duality of the action.

In Sec. 3 we imposed the invariance under T-duality of the junction conditions and we found that we recover standard brane cosmological equations. These are invariant provided that the energy momentum tensor of the matter transforms in an appropriate way. However, this result was achieved by assuming a specific form for the brane Lagrangean. Hence, it would be interesting to explore more general cases, for example by adding a dilatonic potential on the brane and/or without assuming a specific form for the functions $f(\phi)$ and $z(\phi)$ defined in Sec. 2.

Finally, in Sec. 5 we proposed a model where the dual phase of the cosmological equations arises from the bouncing of the brane off of a zero of the effective potential $W(r)$. We showed that we can recover most of the features of the Pre-Big Bang scenario, and that the dual transition can occur at a non-singular point. In particular, we discussed the possibility of modelling a pre-Big Bang superinflationary Universe evolving into a (dual) post-Big Bang accelerating Universe. More in general, the behaviour of the brane cosmological equations are critically influenced by the structure of the bulk (in particular if there are singularities and/or horizons) and of the effective potential. Therefore it is important to study more general solutions to the bulk equations of motion, not only because they are interesting *per se*, but also because they might provide for a realistic brane cosmological model.

Acknowledgements

We wish to thank O. Corradini (Bologna U.) for very helpful suggestions and stimulating discussions, and M. Gasperini (Bari U. & INFN) and G. Veneziano (CERN) for useful comments on the first version.

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